HEAT TRANSFER IN PLANE FILM FORMATION

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A mathematical model is obtained for the process of cooling with formation of a planar film. The solution obtained is verified experimentally.

In formation of a plane film in a polymer phase, transitions occur which cause formation of a so-called primary structure. The structure formed determines the film's response to later oriented extension to a great degree and, in the final reckoning, has a major effect on the quality of the film or film fiber produced [1]. Consequently, a mathematical model of the cooling process during film formation is necessary for directed formation of the primary structure. The cooling kinetics control not only the quality of the product, but also the productivity of the process.

The goal of the present study is to construct a mathematical model of the heat-transfer process in plane film formation.

Preliminary studies have established that the kinematics of elongation of a plane flow in the formation zone are described with sufficient accuracy by the plane section hypothesis proposed in [2, 3]. Moreover, it has been established that at low elongations (as normally occur during formation) the axial velocity distribution is described sufficiently well by a linear law:

$$v_x = v_0 + \Gamma x, \tag{1}$$

where x is the distance from the spinneret; $\Gamma = v_0(K-1)/l$, mean elongation rate gradient; and $K = v_1/v_0$, draw ratio.

Under these conditions the deformation rate tensor has the form

$$\{\dot{\gamma}\} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -0.5 & 0 \\ 0 & 0 & -0.5 \end{bmatrix} \Gamma. \tag{2}$$

A diagram of the forming process is shown in Fig. 1. After completion of the expansion process the melt with a temperature T_0 constant across its section arrives in the formation zone, where it is subject to uniaxial extension. Since expansion is completed in a distance of several millimeters from the spinneret, the extension length is practically equal to the distance from the mouth of the spinneret to the point of tangency with the cooling drum surface. In the elongation segment the film is in thermal interaction with the surrounding air at a temperature of T_c. We locate the origin of the coordinates at the point of greatest expansion (Fig. 1). The z axis is directed along the film width. The cooling conditions permit the following assumptions: $dT/dz = 0$. This fact has been confirmed experimentally. Correspondingly, $d^2T/dz^2 = 0$, i.e., from the viewpoint of heat transfer the problem is a planar one. We neglect thermal conductivity in the axial direction, since $d^2T/dy^2 \gg d^2T/dx^2$. The polymer in the formation zone has a relatively low viscosity and the draw ratio is usually small, so that we neglect dissipative heating.

Under these conditions the boundary-value problem in dimensionless coordinates has the form

$$L(\Theta) = \left(\frac{v_0}{l} + \Gamma X\right)\frac{\partial \Theta}{\partial X} - \frac{\Gamma Y}{2}\frac{\partial \Theta}{\partial Y} - \frac{a}{\delta_0^2}\left(1 + \frac{\Gamma l X}{v_0}\right)\frac{\partial^2 \Theta}{\partial Y^2} = 0; \tag{3}$$

$$X = 0,\ \Theta = 1; \tag{4}$$

$$Y = 0,\quad \frac{\partial \Theta}{\partial Y} = 0; \tag{5}$$

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$$Y = 1, \quad \frac{\partial\Theta}{\partial Y} + \mathrm{Bi}\Theta = \Theta, \tag{6}$$

where $L(\Theta)$ is a differential operator; $\Theta = (T - T_c)/(T_0 - T_c)$, dimensionless temperature; $Y = y\delta_0^{-1}\left(1 + \frac{\Gamma l X}{v_0}\right)^{0.5}$, dimensionless ordinate; $X = x/l$, dimensionless distance; and $\mathrm{Bi} = \alpha\delta_0\lambda^{-1}\left(1 + \frac{\Gamma l X}{v_0}\right)^{0.5}$, Biot criterion. The transverse velocity value in Eq. (3) is obtained by integrating the corresponding component of tensor (2) with consideration of the fact that $Y = 0$, $v_y = 0$. We assume that the heat-transfer coefficient is constant over length and identical for both sides of the film. The problem is symmetric about the X axis, so that it is sufficient to obtain a solution for the region $0 < X < 1$, $0 < Y < 1$.

We find the solution of Eq. (3) by Kantorovich's method, presented in detail in [4, 5]. We seek the solution in the form of the product of two functions [6]:

$$\Theta = \left(\frac{\mathrm{Bi} + 2}{\mathrm{Bi}} - Y^2\right) f(X). \tag{7}$$

The unknown function f(X) is found from the orthogonality condition

$$\int_0^1 L(\Theta)\left(\frac{\mathrm{Bi} + 2}{\mathrm{Bi}} - Y^2\right) dY = 0. \tag{8}$$

The solution in the form of Eq. (7) does not satisfy initial condition (4), so we require its fulfillment on the axis $Y = 0$, whence we obtain the following initial condition for the unknown function:

$$f(0) = \frac{\mathrm{Bi}}{\mathrm{Bi} + 2}.$$

In the formation process the Biot criterion has small numerical values ($\mathrm{Bi} < 0.1$), as a consequence of which the temperature drop across the film thickness will be small. Therefore, it can be assumed that the temperature across the section is constant and equal to the temperature on the axis. In this case the final solution has the form

$$-\mathrm{Gz}^* \ln\bar{\Theta} = 2\,\mathrm{Bi}_0(\sqrt{V} - 1) - \frac{\mathrm{Bi}_0^2}{3}\ln\left[\frac{60V + 40\,\mathrm{Bi}_0\sqrt{V} + 8\,\mathrm{Bi}_0^2}{60 + 40\,\mathrm{Bi}_0 + 8\,\mathrm{Bi}_0^2}\right] + \frac{13\sqrt{5}\,\mathrm{Bi}_0^2}{36}\left\{\operatorname{arctg}\left[\frac{\sqrt{5}(3\sqrt{V} + \mathrm{Bi}_0)}{\mathrm{Bi}_0}\right] - \operatorname{arctg}\left[\frac{\sqrt{5}(3 + \mathrm{Bi}_0)}{\mathrm{Bi}_0}\right]\right\}, \tag{9}$$

where $\mathrm{Gz}^* = \Gamma\delta_0^2/a$ is the modified Graetz criterion; $V = 1 + (\Gamma l X/v_0)$, dimensionless velocity; and $\mathrm{Bi}_0 = \alpha\delta_0/\lambda$, Biot criterion, calculated for the initial film thickness. The solution in the form of Eq. (9) can be represented as a nomogram, describing the dependence of the left side of the equation upon the dimensionless velocity and the Biot criterion, which significantly simplifies calculation of the temperature of the film formed.

The solution obtained was verified experimentally. A film was formed from polypropylene (IR-3.5). The spinneret width was 0.16 m; the gap between the jaws was 0.0005 m. Initial velocity (in the expansion section) was determined by the tracer method, since it was found that the velocity was practically constant to a distance of 4 cm. Formation occurred vertically downward. Distance from the spinneret jaws to the point of tangency with the cooling drum surface was 0.17 m. Initial melt temperature was in the range 230-260°C. Formation conditions were varied within the following limits: v_0 = 0.0071-0.0167 m/sec; v_1 = 0.014-0.098 m/sec; K = 1.97-11.75; δ_0 = 0.000465-0.000925 m; T_c = 20°C.

Temperature was measured by a low-inertia microthermocouple at various distances from the spinneret jaws in 0.03-m steps. In the calculations the thermophysical properties of polypropylene were taken at their average values for the 200-250°C temperature range.

The heat-transfer coefficient was determined from the expression

$$\alpha = \alpha_1 + \alpha_2 + \alpha_3, \tag{10}$$

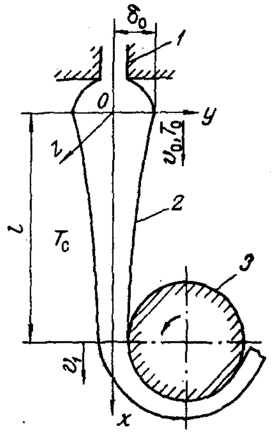

Fig. 1

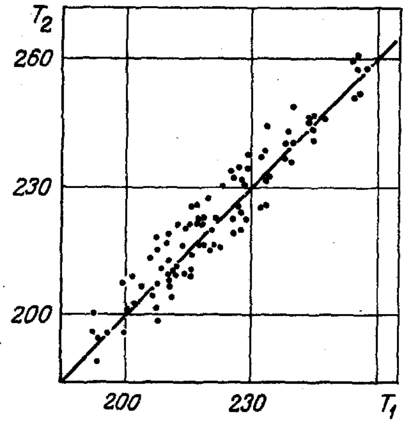

Fig. 2

Fig. 1. Diagram of the plane film formation process: 1) spinneret; 2) film; 3) cooling drum.

Fig. 2. Comparison of experimental T_2, °C, and calculated T_1, °C, film temperatures in formation zone.

where α_1 is the heat-transfer coefficient produced by radiation, α_2 is the heat-transfer produced by free convection [7], and α_3 is the heat-transfer produced by forced convection [8]. All three components of the heat-transfer coefficient were taken as averaged over flow length. The quantity α_1 depends on film thickness [9]. The mean film thickness in the formation zone was determined by the formula

$$\delta_m = \frac{\delta_0(\sqrt{K}-1)}{\sqrt{K}\ln\sqrt{K}}.$$

The mean velocity value needed to calculate α_3 was defined by the expression

$$v_m = \frac{v_0(K-1)}{\ln K}.$$

The value of the total heat transfer coefficient was found to be within the range 15-29 W/m^2·deg K. On the average, the values of the coefficients appearing in Eq. (10) were in the ratio $\alpha_1:\alpha_2:\alpha_3$ = 50%:40%:10%.

Figure 2 compares experimental and calculated film temperatures in the formation segment. The divergence between experimental and theoretical results lies within the limits of experimental uncertainty. Consequently, the mathematical model obtained here may be used for calculation of film cooling in the formation zone.

NOTATION

Γ, mean axial velocity gradient; v_x, current axial velocity; v_0, initial polymer velocity; v_1, sampling velocity; K, draw ratio; $\{\dot{\gamma}\}$, deformation rate tensor; x, y, z, spatial coordinates; X, Y, dimensionless coordinates; $L(\Theta)$, differential operator; T, temperature; T_0, initial temperature; T_c, temperature of surrounding medium; Θ, dimensionless temperature; $\bar{\Theta}$, dimensionless temperature averaged over film thickness; a, thermal-diffusivity coefficient; $2\delta_0$, initial film thickness; λ, thermal conductivity; α, heat-transfer coefficient; f(X), distance function; Bi, Biot criterion, Bi_0, Biot criterion calculated for initial film thickness; Gz*, modified Graetz criterion; V, dimensionless velocity; α_1, α_2, α_3, heat-transfer coefficients produced by radiation, free convection, and forced convection, respectively; v_c, δ_c, mean velocity and film half-thickness in formation zone; T_1, calculated temperature value; T_2, experimental temperature value; l, formation zone length.

LITERATURE CITED

1. L. A. Polovikhina, Khim. Volokna, No. 4 (1972).
2. N. V. Tyabin et al., Plast. Massy, No. 4 (1977).
3. V. G. Litvinov, Mekh. Polim., No. 2 (1967).
4. L. V. Kantorovich and V. I. Krylov, Approximate Methods in Higher Analysis [in Russian], Fizmatgiz, Moscow (1962).

5. A. V. Lykov, Izv. Akad. Nauk SSSR, Énerg. Transport, No. 5 (1970).
6. P. V. Tsoi, Methods for Calculation of Individual Heat- and Mass-Transfer Problems [in Russian], Énergiya, Moscow (1971).
7. M. A. Mikheev and I. M. Mikheeva, Fundamentals of Heat Transfer [in Russian], Énergiya, Moscow (1977).
8. A. V. Lykov, Heat and Mass Transfer (Handbook) [in Russian], Energiya, Moscow [1971).
9. W. Predöhl, Kunststoffe, 61, No. 7 (1971).

NUMERICAL SIMULATION OF AN EXPLOSIVE PLASMA GENERATOR IN THE GASDYNAMIC APPROXIMATION

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A two-dimensional gasdynamic model of a plasma generator is proposed. A numerical solution of the problem is obtained, and peculiarities of the gasdynamic flow are considered. Results are compared with experiment.

The explosive plasma generator proposed by A. E. Voitenko is used to produce dense plasma and high-velocity plasma flows which convert a large portion of their energy into radiant energy. The generator (Fig. 1a) is a closed chamber in the form of a spherical segment, filled with the working gas. A metal plate driven by the explosion products moves toward the top of this segment, forcing the working gas into the tube. Compressed many times by the action of the shock wave, the gas is strongly accelerated along the device axis, with the mass velocity of the gas being close to the phase velocity of the motion of the point where plate and spherical segment join, and many times higher than the velocity of the plate motion. As a result, the gas is transformed into a plasma with parameters $T \sim 10^{5}$°K, $p \sim 10^{10}$ Pa, $N \sim 10^{27}$ m^{-3}. Directly upon exit from the tube the plasma occupies a small volume; then there occurs nonstationary ejection of a flare, whereupon the jet velocity may reach values of $\sim(40-90) \times 10^{3}$ m/sec, depending on the nature of the working gas and conditions in the tube [1-4]. The continuous spectrum radiation from the front of the jet has a flux density of $\sim 10^{10}-10^{12}$ W/m^2 with a total light energy of $\sim 3 \cdot 10^{5}$ J/m^2 over an irradiation time on the order of tens of μsec [9]. These and a number of other desirable features of such a generator have stimulated the appearance of a large number of experimental studies in which both the gasdynamic and radiant characteristics of the plasma flow produced have been studied [1-10]; however, obvious difficulties have prevented study of the dynamics of the processes occurring within the device. As far as development of a theory of processes in the generator is concerned, we have only [3], in which certain considerations pertaining to the principle of device operation were set forth and estimates of plasma parameters made, together with [6], which presented certain calculated and experimental data. In [6] the flow in the generator compression chamber was assumed two-dimensional, while the flow of plasma into the tube, forming the directed flow, was assumed one-dimensional. Results were presented only for certain integral plasma parameters and shock-wave trajectories in the generator, and no data characterizing the development of gasdynamic processes were presented.

The present study is an attempt at direct numerical simulation of the gasdynamic processes in such a generator by indirect calculation of the nonstationary axisymmetric flow which develops throughout its entire volume. We will use the following formulation of the problem. At the initial moment t = 0 under the action of the explosion products the plate of mass M begins to move into the compression chamber (the spherical segment) at a velocity V, losing its kinetic energy to compression and acceleration of the working gas filling the segment in accordance with the expression

$$\frac{MV^2}{2} = E_{\mathrm{in}} - \int_0^t \Delta pSU dt'. \qquad (1)$$

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